SM3 5.2 Solve Rational Exponent Equations

Today, we explore solving equations that contain all root expressions, like $\sqrt[3]{x}$, $\sqrt[4]{2x-1}$, or $x^{\frac{2}{3}}$.

In an equation that contains just one root, our plan is to isolate the root, remove the root by powering, and then continue solving using techniques from previous units. Unfortunately, because some domains of root functions are not all real numbers, we sometimes need to check for extraneous solutions. In the event that we find an extraneous solution, we reject that particular solution.

Solve: $\sqrt[3]{x-5}-4=2$

$\sqrt[3]{x-5}-4=2$	Given
$\sqrt[3]{x-5}=6$	Add 4 to isolate the root
$x-5=216$	Cube both sides to remove the root
$x=221$	Continue solving the equation

Check: The domain of a root with odd index is all real numbers, so there's no need to check.

We'll keep our answer of $x=221$.

Fractional exponents are a convenient way to have both a root and a power represented. For example, $x^{2/3}$ means both $\sqrt[3]{x^2}$ or $\left(\sqrt[3]{x}\right)^2$. To remove a fractional exponent from a variable, you need to use a root to remove the power and also a power to remove the root. Both can be accomplished at the same time by selecting a fractional exponent for both sides that is the reciprocal of the original.

Solve: $x^{3/4}=27$

$x^{3/4}=27$	Given
$\left(x^{3/4}\right)^{4/3}=(27)^{4/3}$	Raise both sides to the reciprocal of the original power
$x=(27)^{4/3}$	The left exponents multiply to 1
$x=81$	The cube root of 27 is 3, and 3^4 is 81

When we solved $x^2=9$ by square rooting in SM2, we needed a $\pm$ symbol to find both solutions. When you use an even-powered root on both sides, you'll need the same $\pm$ symbol.

Solve: $x^{2/3}=4$

$x^{2/3}=4$	Given
$\left(x^{2/3}\right)^{3/2}=(4)^{3/2}$	Raise both sides to the reciprocal of the original power
$x=(4)^{3/2}$	The left exponents multiply to 1
$x=\pm 8$	The square root of 4 is ± 2, and 2^3 is 8 while $(-2)^3$ is -8

HW6.2

Solve each equation for x over the set of real numbers.

1) $\sqrt[3]{x}-8=-3$

2) $\sqrt[3]{x-12}=3$

3) $\sqrt[4]{x+2}=2$

4) $\sqrt[3]{12x}+10=7$

5) $\sqrt[5]{x-3}=2$

6) $\sqrt[3]{7-x}+2=7$

7) $2\sqrt[4]{x}=-12$

8) $5\sqrt[3]{x+6}-1=24$

9) $16\sqrt[6]{x-3}=32$

10) $2x^{2/3}=32$

11) $x^{2/5}-1=8$

12) $x^{3/2}=-125$

13) $(3x)^{4/3}+2=83$

14) $\frac{1}{12}(x-5)^{1/2}=3$

15) $\frac{1}{4}(x-2)^{3/2}=16$

16) $(x+1)^{3/7}=27$

17) $(5x-26)^{5/6}=32$

18) $\frac{1}{2}(2x+4)^{10/3}=512$